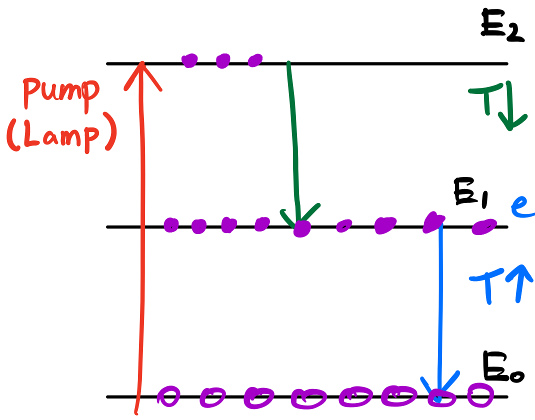


T↓ Short (e^-'s in this state drop fast)

excited state

T↑ Long (e^-'s stay here for a long time)

Speed of Light $= \frac{c}{e_i}$ → index of refraction

Index of Refraction (Decreases with)

① Material Dispersion

② Waveguide Dispersion

- Small wavelengths → light is in the silicon. Higher index
- Long Wavelengths → light is avg'd more over material outside the silicon and therefore the index is lower

Fabry - Perot Cavity:

- 2 mirrors where light bounces back and forth
- Some material in between the mirrors
- Can be Horizontal or Vertical
- Mirrors (Bragg Gratings)

Wave Equations

$$\nabla^2 \boldsymbol{E} - \mu\epsilon \frac{\partial^2}{\partial t^2}\boldsymbol{E} = 0 \quad \text{(Maxwell)}$$

solution:

$$\boldsymbol{E} = E_o \boldsymbol{u} e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}$$

$\mathbf{E_o}$ is the amplitude of the wave.
ω is the frequency.
$\boldsymbol{k}$ is the wavevector with a propagation constant $k = |\boldsymbol{k}|$; $\boldsymbol{u}$ is perpendicular to the direction of propagation.

- The magnitude of the wavevector is

$$\beta = |\boldsymbol{k}| = \omega\sqrt{\mu\epsilon}$$

- The phase velocity is:

$$\nu = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{n\sqrt{\mu_0\epsilon_0}}$$

where n is the index of refraction.

- The wavelength in the material is:

$$\lambda' = \frac{2\pi}{k}$$

Energy of a Wave:

- The time-averaged energy flux (Poynting's theorem) is

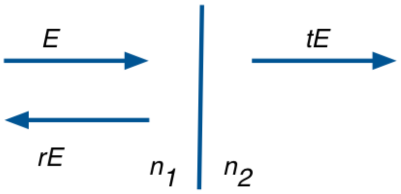
$$\boldsymbol{S} = \frac{1}{2\eta}|E_0|^2\boldsymbol{u}_3 = \frac{\boldsymbol{k}}{2\omega\mu}|\boldsymbol{E}|^2$$

- The time-averaged energy density is

$$U = \frac{1}{2}\epsilon|E_0|^2 = \frac{1}{2}\epsilon|\boldsymbol{E}|^2$$

Reflections at Dielectric Interfaces

- Consider the plane wave E, *normally* incident on a dielectric interface:



- The reflection coefficient, r, is

$$r_{12} = \frac{n_1 - n_2}{n_1 + n_2}$$

- The transmission coefficient, t, is

$$t_{12} = \frac{2n_1}{n_1 + n_2}$$

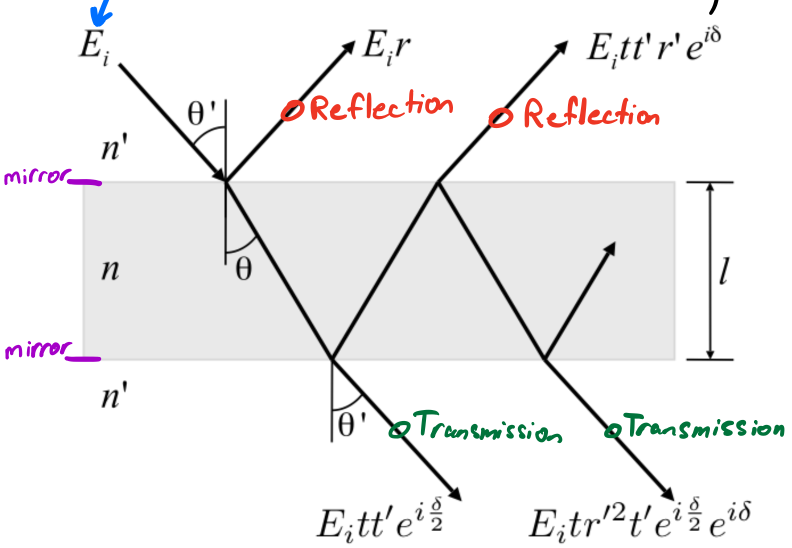
- Note that if light was originating in the n_2 medium, the reflection coefficient would be of opposite sign, i.e. $r_{21} = -r_{12}$.
- The power reflection coefficient is $R = r^2$.
  The power transmission coefficient is $T = t_{12}t_{21}$.
- Conservation of energy tells us that

$$T + R = 1$$

Fabry-Perot Cavity

External Light Source

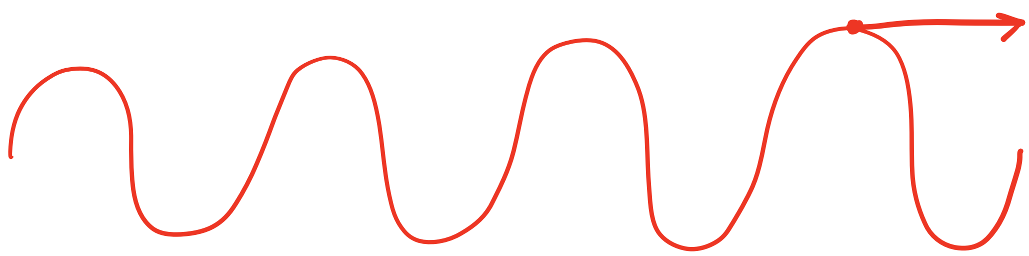
$e^{i\delta} = 1$, Max Reflectivity
↳ $\delta = 0, 2\pi, 4\pi, \dots$



- The Fabry-perot etalon is the most basic optical resonator. It consists of a plane-parallel plate of thickness l and index n that is immersed in a medium of index n'.
- r = reflection coefficient for waves from n' toward n.
- r' = reflection coefficient for waves from n toward n'.
- t, t' = transmission coefficients
- Assume normal incidence, $\theta = 0$
- Round-trip phase shift: $\delta = k\Delta = \underbrace{-\frac{2\pi}{\lambda}n}_{k} \cdot \underbrace{2l}_{\Delta}$

Phase vs. Group Velocity

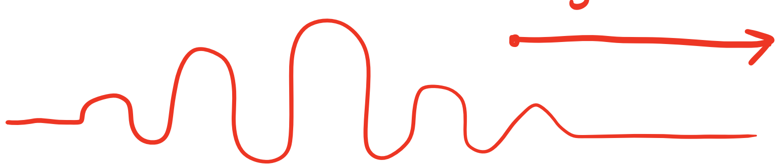
V_p (Phase velocity)



$f = 200$ pHz

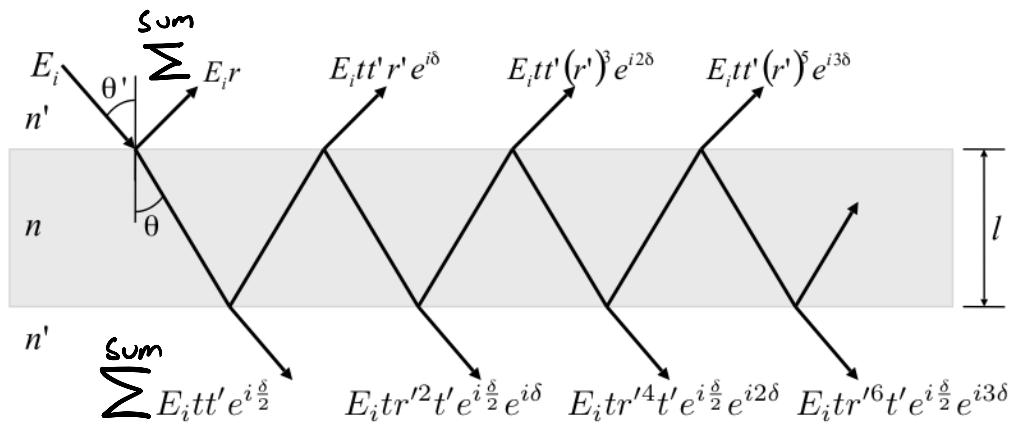
Multiply by a pulse. ie: Grab a "group" of this sinusoid

V_g (Group Velocity)



Fabry-Perot Model - Multiple Beam Interference

Light is incident from the top, E_i, transmitted through bottom, E_t.



We can express the transmitted field as:

$$\sum E_t = E_i t t' e^{i\frac{\delta}{2}} + E_i t t' (r')^2 e^{i\delta} e^{i\frac{\delta}{2}} + E_i t t' (r')^4 e^{i2\delta} e^{i\frac{\delta}{2}} + E_i t t' (r')^6 e^{i3\delta} e^{i\frac{\delta}{2}} + \ldots$$

- Both sums tell us what comes out (Both reflection and Transmission Components)
- δ is the "round-trip" phase change

The transmitted field, E_t, simplifies to:

$$\begin{aligned} E_t =& E_i t t' e^{i\delta/2} \left[1 + (r')^2 e^{i\delta} + (r')^4 e^{i2\delta} + \ldots\right] \\ =& E_i \left[\frac{t t' e^{i\delta/2}}{1 - (r')^2 e^{i\delta}}\right] \quad \text{(infinite geometric series)} \\ =& \boxed{E_i \left[\frac{T e^{i\delta/2}}{1 - R e^{i\delta}}\right]} \end{aligned}$$

No mirrors $\rightarrow R=0$

100% Reflectivity $\rightarrow R=1$

The normalized transmission is:

$$\frac{E_t}{E_i} = \frac{T e^{i\delta/2}}{1 - R e^{i\delta}}$$

And the power transmitted (light intensity):

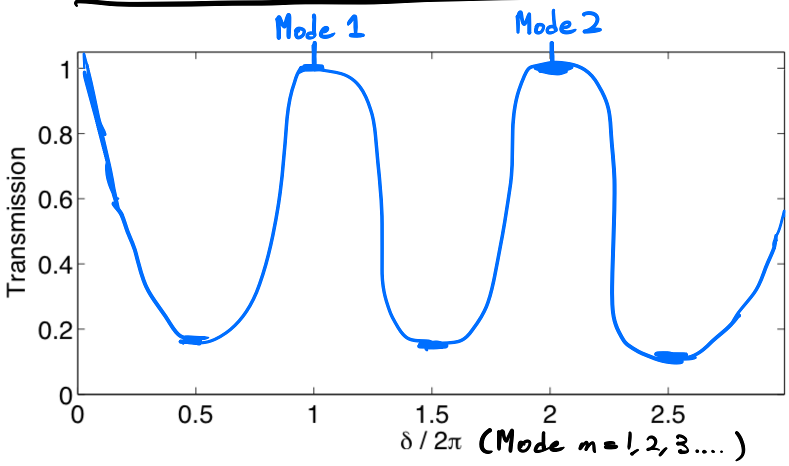
Transmission

$$\frac{I_t}{I_i} = \frac{E_t E_t^*}{E_i E_i^*} = \frac{T^2}{(1 - R e^{i\delta})(1 - R e^{-i\delta})} = \boxed{\frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(\delta/2)}}$$

Resonance (Light goes straight through) :

- When $\sin^2(\delta/2) = 0 \rightarrow I_t/I_i = 1$ → Resonance Condition
  (i.e. all of the light is transmitted – no reflection).
- Since $\sin^2(\delta/2)$ is periodic, there are many solutions which yield 100% transmission. These wavelengths are the Fabry-Perot modes, or Fabry-Perot resonant frequencies.
- This occurs for $\sin^2(\delta/2) = 0 \rightarrow \delta/2 = m\pi = \boxed{\frac{2\pi n l}{\lambda_m},\ (m = 1, 2, 3...)}$
- The solutions are λ_m, or in frequency, $\nu_m = \frac{c}{\lambda_m} = m\frac{c}{2nl}$
- As we will see later laser will oscillate at one of these wavelengths

Transmission Vs. Mode Plot:



$\frac{\delta}{2\pi} = 0.5 \rightarrow \frac{\pi}{2}$

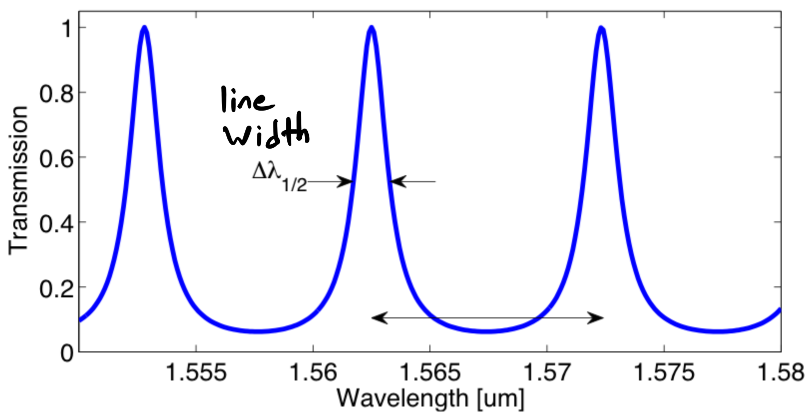
$\frac{0.5}{0.5+1.2} =$

- (R=0.3)

  - The free spectral range (FSR) is the spacing between adjacent modes of the filter, and is:

$$\Delta\nu = \nu_m - \nu_{m-1} = \frac{c}{2nl}$$

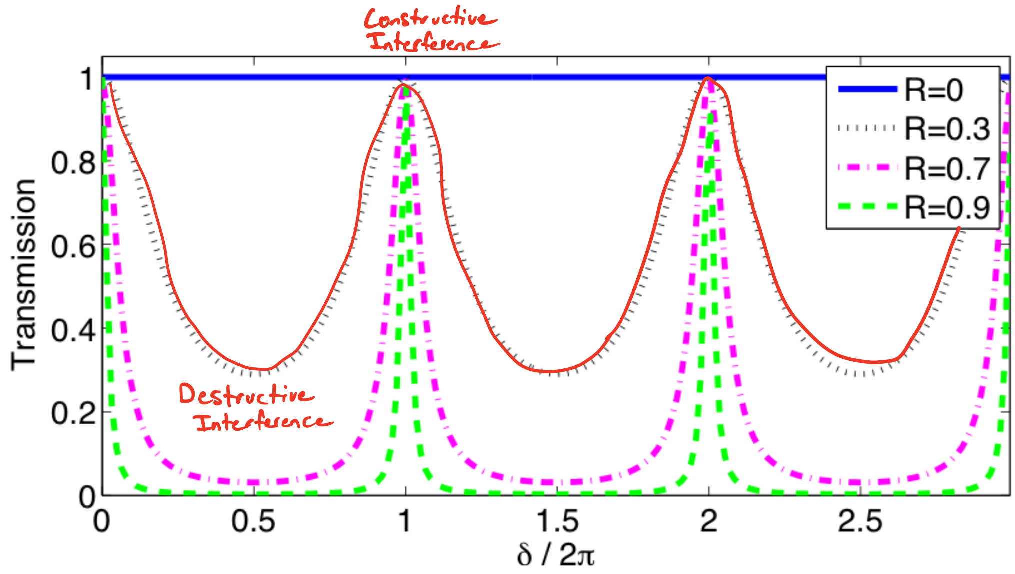
-Long Resonator → Peaks are spaced close together
ie: ↑L ↓Δν



- The linewidth ($\Delta\lambda_{\frac{1}{2}}$) of the resonator is defined as the full-width half-max (FWHM) of the transmission spectrum:

$$\Delta\lambda_{\frac{1}{2}} = \lambda_2 - \lambda_1$$

- (R=0.6)



- The linewidth of the resonator changes with the reflection coefficient
- A larger reflection leads to a narrower linewidth

Fabry-Perot Model - Reflection

The reflected field, E_r, simplifies to:

$$E_r = E_i r + E_i t t' r' e^{i\delta}\left[1 + (r')^2 e^{i\delta} + (r')^4 e^{i2\delta} + \ldots\right]$$
$$= E_i\left[r + \frac{t t' r' e^{i\delta}}{1-(r')^2 e^{i\delta}}\right] \quad \text{(infinite geometric series)}$$
$$= E_i\left[r - \frac{T r e^{i\delta}}{1-R e^{i\delta}}\right], \quad r = -r'$$

The normalized reflection is:

$$\frac{E_r}{E_i} = r - \frac{T r e^{i\delta}}{1-R e^{i\delta}} = \frac{r(1-e^{i\delta})}{1-R e^{i\delta}}$$

And the power reflected (light intensity): Reflection

$$\frac{I_r}{I_i} = \frac{E_r E_r^*}{E_i E_i^*} = \boxed{\frac{4R\sin^2(\delta/2)}{(1-R)^2 + 4R\sin^2(\delta/2)}}$$

Fabry-Perot Model - with Loss

Power

If we consider loss in the cavity, where the per-pass intensity gain (or loss) is

$$A = a^2 = e^{-\alpha l}$$

and the per-pass field amplitude gain (or loss) is

Electric field

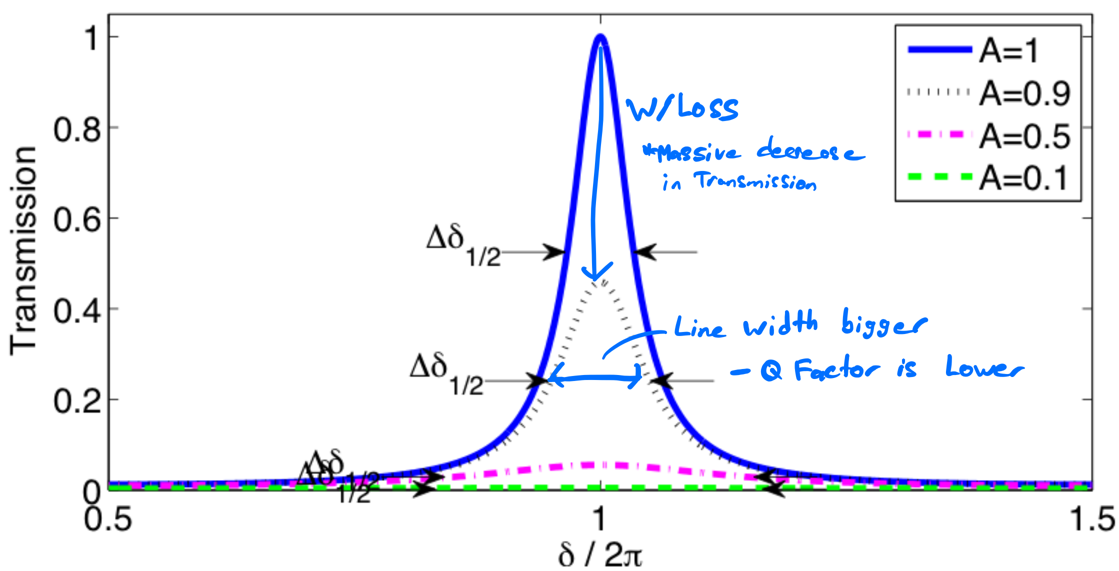
$$a = e^{-\frac{\alpha}{2} l}$$

The transmitted field, E_t, becomes:

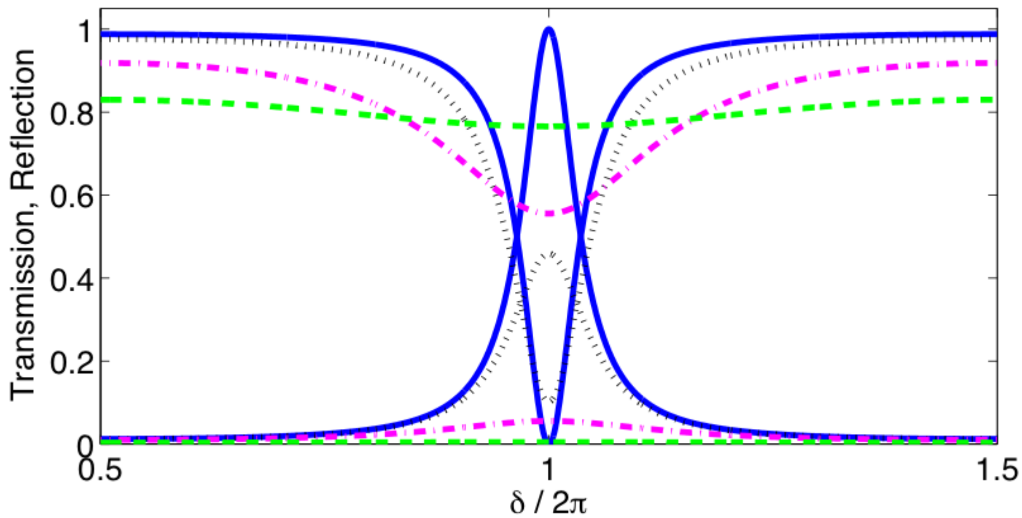
$$\begin{aligned} E_t =& E_i t t' a e^{i\delta/2}\left[1 + (ar')^2 e^{i\delta} + (ar')^4 e^{i2\delta} + ...\right] \\ =& E_i \left[\frac{tt'ae^{i\delta/2}}{1-(ar')^2 e^{i\delta}}\right] \quad \text{(infinite geometric series)} \\ =& E_i \left[\frac{Tae^{i\delta/2}}{1-RAe^{i\delta}}\right] \end{aligned}$$

The normalized power transmitted (light intensity) is: Transmission W/Loss

$$\frac{I_t}{I_i} = \frac{E_t E_t^*}{E_i E_i^*} = \boxed{\frac{(1-R)^2 A}{(1-AR)^2 + 4AR\sin^2(\delta/2)}}$$



- Transmission spectrum amplitude decreases with increasing losses.
- Linewidth broadens.
- R=0.8



- Reflection spectrum dip decreases with increasing losses.
- R=0.8

Loss (where does it come from):

- Material absorbtion of light
- Increase Width of waveguide → Better Q Factor

Fabry-Perot - Linewidth

- The full-width half-max linewidth is found by finding the half-power point, $\delta_{\frac{1}{2}}$:

$$T(\delta_{\frac{1}{2}}) = \frac{1}{2}T(0)$$

$$\frac{(1-R)^2 A}{(1-AR)^2 + 4AR\sin^2(\frac{\delta_{\frac{1}{2}}}{2})} = \frac{1}{2}\frac{(1-R)^2 A}{(1-AR)^2}$$

$$\delta_{\frac{1}{2}} = 2\sin^{-1}\frac{1-AR}{2\sqrt{AR}} \approx \boxed{\frac{1-AR}{\sqrt{AR}}}$$

The FWHM is:

$$\Delta\delta_{\frac{1}{2}} = 2\delta_{\frac{1}{2}} = 2\frac{1-AR}{\sqrt{AR}}$$

$$\text{since: } \delta = \frac{2\pi}{\lambda}n2l = \frac{\omega n 2l}{c}$$

$$\Delta\omega_{\frac{1}{2}} = 2\frac{1-AR}{\sqrt{AR}}\frac{c}{2nl}, \quad \Delta\nu_{\frac{1}{2}} = \boxed{\frac{1-AR}{\pi\sqrt{AR}}\frac{c}{2nl}}$$

Fabry-Perot Model - Finesse

- A measure of line-width compared to the spacing of the modes
  ie: the sharpness of the resonator

- The finesse ($\mathcal{F}$) of a resonator is defined as the ratio of the FSR to resonance linewidth:

$$\mathcal{F} = \frac{FSR}{\Delta\lambda_{\frac{1}{2}}} = \frac{\Delta\nu}{\Delta\nu_{\frac{1}{2}}}$$
$$= \frac{\frac{c}{2nl}}{\frac{1-AR}{\pi\sqrt{AR}}\frac{c}{2nl}}$$
$$= \boxed{\frac{\pi\sqrt{AR}}{1-AR}}$$

- This is a measure of the sharpness of a resonance relative to the mode spacing.

Fabry-Perot Model - Q (Quality Factor)

- The resonator Quality factor, Q, is a measure of the sharpness of the filter relative to the central frequency, and defined as ($\mathcal{E}$ is stored energy):

$$\boxed{Q = \omega\frac{\mathcal{E}}{d\mathcal{E}/dt}}$$

- Note: This expression works very well to estimate Q from FDTD simulations. (Plot the intensity versus time.)
- For this, we need to calculate the total losses, both from the mirrors and propagation losses (absorption, scattering).
- Per-pass intensity loss $= 1 - AR$
- Distributed total loss, per unit length, α_{tot}:

$$A_{tot} = AR = e^{-\alpha l}R = e^{-\alpha_{tot}l}$$

$$\alpha_{tot} = \alpha - \frac{1}{l}\ln R$$

Fabry-Perot Model - Photon Lifetime

- Total loss in cavity = Mirror loss + absorption, $\alpha_{\text{tot}} = \alpha_m + \alpha$.
- The loss α_{tot} $[cm^{-1}]$ describes the loss over distance,

$$P(z) = P(0)e^{-\alpha_{\text{tot}} z}$$

- However, we would like to model the time behaviour of a resonator,

$$\frac{dP}{dt} = \frac{dP}{dz}\frac{dz}{dt}$$

$$\frac{dP}{dz} = -\alpha_{\text{tot}} P, \quad \frac{dz}{dt} = \frac{c}{n}$$

multiplying, get $\frac{dP}{dt} = -\alpha_{\text{tot}} P \frac{c}{n} = -\frac{P}{\tau_p} = R_{\text{loss}}$

where $\tau_p = \left[(\alpha + \alpha_m)\frac{c}{n}\right]^{-1}$ is the photon lifetime
(1 / Rate of photon decay = average time spent in the cavity)

Q: Calculate the photon lifetime for $\alpha_{\text{tot}} = 10cm^{-1}, n = 3$.

Fabry-Perot Model - Q

- The decay of the energy in the cavity is:

$$\frac{d\mathcal{E}}{dt} = -\frac{\mathcal{E}}{\tau_p}$$

- So Q is:

$$Q = \omega \frac{\mathcal{E}}{d\mathcal{E}/dt} = \omega \tau_p$$

- Q is also defined as:

$$Q = \frac{\omega}{\Delta\omega_{\frac{1}{2}}}$$

- The two definitions give equal results for “small-enough” losses, i.e. large Q values. This occurs when $-\ln R \approx \frac{1-R}{\sqrt{R}}$.
- The field decays to $1/e$ in time τ_p. Thus, Q is the number of optical field oscillations before the field decays to $1/e$.

- Finesse:

$$\mathcal{F} = \frac{\pi\sqrt{AR}}{1-AR}$$

- This can be related to the total loss α via $\ln R$.

$$\mathcal{F} \approx \frac{\pi}{\alpha l}$$

- We can find the field decaying to $1/e$ when:

$$\frac{1}{e} = e^{-1} = e^{-\alpha l 2N}$$

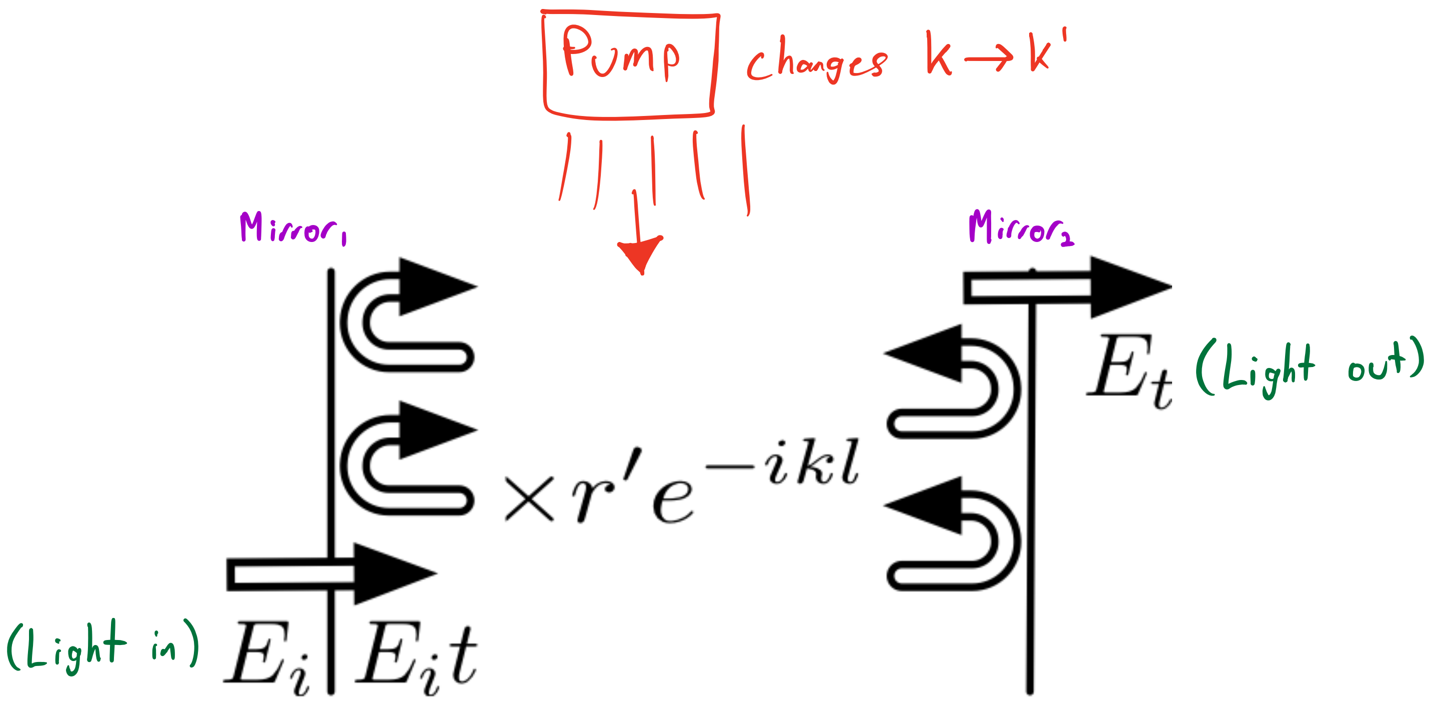
where N is the number of round trips in the resonator.

$$-\alpha l 2N = -1, \quad \frac{\pi}{F} 2N = 1, \quad N = \frac{\mathcal{F}}{2\pi}$$

- Hence, finesse$/2\pi$ describes the average number of oscillations the field lives in the cavity.

Fabry-Perot Laser

- A laser is constructed using a Fabry-Perot cavity.
- For each pass in the cavity, the field is modified (phase and amplitude) by the factor $\times r' e^{-ikl}$.



- The propagation constant is modified by providing optical gain.

$$k' = k + \Delta k + i\underbrace{\frac{\gamma}{2}}_{\text{Gain}} - i\underbrace{\frac{\alpha}{2}}_{\text{Loss}}$$

where
  - k is the original propagation constant
  - Δk is the change in propagation constant (due to active atoms)
  - γ is the optical gain why factor of $\frac{1}{2}$?
  - α is the optical loss
- Note: All these parameters are wavelength dependant. Optical gain, $\gamma(\lambda)$, is typically approximately a Gaussian function, with a bandwidth of $\sim 100nm$ in semiconductors.

- Similar to an RF oscillator, a laser will begin to oscillate when the round-trip provides unity gain.
- From the diagram, the round trip gain is: (two reflections, and a path length of $2l$)

$$\begin{aligned} (r')^2 e^{-ik'2l} &= 1 \\ r^2 e^{-i\left(k+\Delta k + i\frac{\gamma}{2} - i\frac{\alpha}{2}\right)2l} &= 1 \\ r^2 e^{-i(k+\Delta k)2l} e^{\left(\frac{\gamma}{2} - \frac{\alpha}{2}\right)2l} &= 1 \end{aligned}$$

- The real part is: $r^2 e^{\left(\frac{\gamma}{2} - \frac{\alpha}{2}\right)2l} = 1$. This is the amplitude condition, where the field returns with the same amplitude after a round trip.
- The imaginary part is: $e^{-i(k+\Delta k)2l} = 1$. This is the phase condition, where the field must return with the same phase (or a multiple of 2π). $2m\pi = (k + \Delta k)\, 2l$, $(m = 1, 2, 3 \ldots)$

Laser Threshold

- The amplitude condition can be solved to provide the minimum gain required for the laser to operate. This is called the threshold gain, γ_t.

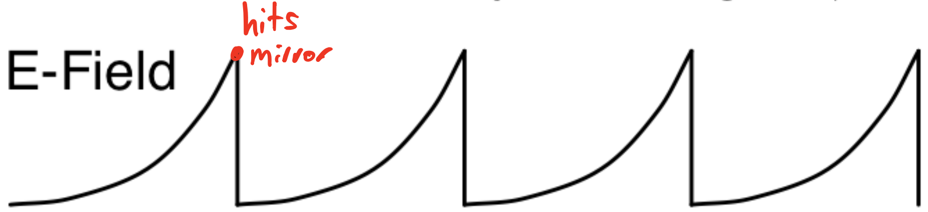
$$r^2 e^{\left(\frac{\gamma}{2}-\frac{\alpha}{2}\right)2l} = 1, \quad R = r^2$$

$$\ln(R) + \left(\frac{\gamma}{2} - \frac{\alpha}{2}\right) 2l = 1$$

$$\boxed{\gamma_t = \alpha - \frac{1}{l}\ln(R)}$$

gain = loss

- Thus, the laser will "turn on" only when the gain, $\gamma_o \geq \gamma_{th}$



Summary

The optical cavity (e.g. Fabry-Perot) is important for lasers because:

- The cavity modes determine the possible lasing wavelengths
- The cavity losses (mirror loss, internal losses) determine the laser threshold